MODEL QUESTIONS
For Mathematics GE 2
Semester 2

Short Answer Questions

1. If u= log ($x^3 + y^3 + z^3 - 3xyz$), show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-g}{(x+y+z)^2}$

2. If $= sin^{-1}\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$, Show that $\frac{\partial u}{\partial x} = -\frac{y}{x}\frac{\partial u}{\partial y}$.

3. If u= $\sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

4. Find the radius of curvature at any point of the parabola $y^2 = 4ax$ and deduce that the radius of curvature at parabola is equal to semi-latus rectum.

5. Find the radius of curvature for the curve, s= c tan ϕ at point (s, ϕ).

6. Prove that $\int_0^{\frac{\pi}{2}} \frac{sin2\theta}{sin^4\theta+cos^4\theta}\, d\theta = \frac{\pi}{2}$.

7. Find the radius of curvature at any point for the curve, $r = ae^{m\theta}$

8. Calculate Divergence of the vector field:

$$\vec{f} = \frac{\vec{x}}{x+y+z}\, i + \frac{\vec{y}}{x+y+z}\, j + \frac{\vec{z}}{x+y+z}\, k$$

9. Evaluate $\int_0^a \frac{x^2}{(a^2-x^2)^{1/2}}\, dx$

10. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{cosx}}{\sqrt{cosx}+\sqrt{sinx}}\, dx$

11. Evaluate $\lim_{n\to\infty} \left\{\left(\frac{n}{n^2+1^2}\right)\left(\frac{n}{n^2+2^2}\right) + \cdots + \left(\frac{n}{2n^2}\right)\right\}$

12. Evaluate $\int_0^{\frac{\pi}{4}} tan^5 x dx$ using reduction formula

13. Calculate the divergence of the vector field

$\vec{f} = \frac{z-x}{x^2+y^2+z^2}\, \vec{i} + \frac{x-y}{x^2+y^2+z^2}\, \vec{j} + \frac{y-z}{x^2+y^2+z^2}\, \vec{k}$

14. If $\vec{r} =$ x $\vec{i}$ + y $\vec{j}$ + z $\vec{k}$ and $\vec{a}$ is a constant, prove that, Div. $[\vec{a} \times (\vec{r} \times \vec{a})] = 2a^2$

Long Answer Questions

1. If u is a homogeneous function of nth order in x, y, z such that u = x^n f $\left(\frac{y}{x}, \frac{z}{x}\right)$, then Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ = nu

2. If **ρ** be the radius of curvature of a parabola at a point whose distance along curve from fixed point is *s* then $3\rho\frac{\partial^2\rho}{\partial s^2} - \left(\frac{\partial\rho}{\partial s}\right)^2 - 9 = 0.$

3. If u be a homogeneous function of nth degree in x, y, z and if u =f (X,Y,Z) where X,Y,Z are differential coefficients of 'u', w.r.t. x, y, z respectively then Prove that, $X\frac{\partial f}{\partial X} + y\frac{\partial f}{\partial Y} + z\frac{\partial f}{\partial Z} = \frac{n}{n-1}$ u

4. Write the working rule to find the asymptotes on the algebraic curve and find real asymptotes of the curve $x^3 + y^3 = 3axy$.

5. Obtain the polar tangential formula for radius of curvature.

6. If r=a sec 2θ, Prove that $\rho = \frac{r^4}{3p^3}$

7. Prove that $\int_0^{\frac{\pi}{2}} cos^{n-2}xsinnxdx = \frac{1}{n-1}$

8. Evaluate $\int_\alpha^\beta \sqrt{(x-\alpha)(\beta-x)}$ dx

9. Evaluate

a. $\int_0^{\frac{\pi}{2}} \frac{cosx}{(1+sinx)+(1+cosx)}dx.$

b. $\int_0^{\frac{\pi}{2}} \frac{x^2}{(xsinx+cosx)^2}dx.$

10. Evaluate $\lim_{n\to\infty} \left\{\left(1+\frac{1}{n^2}\right)\left(1+\frac{4}{n^2}\right)\left(1+\frac{9}{n^2}\right)\ldots\ldots(2)\right\}^{1/n}$

11. Evaluate $\int_0^{\frac{\pi}{2}} \sin^n xdx$

12. Evaluate $\int_0^{\frac{\pi}{2}} \sin^m x\cos^n xdx$ such that m, n>0 and m, n ϵ I

13. Prove that grad $\left(\frac{f}{g}\right) = \frac{g\,(grad\,f) - f\,(grad\,g)}{g^2}$

14. Find the derivative of ***f*** at ***P*** in the direction of $\overrightarrow{a}$ where $f = e^{yt}cosx + e^{zx}cosy + e^{xy}cosz$ $P\left(\frac{\pi}{6}, \frac{\pi}{3}, 0\right)$, $\overrightarrow{a} = 3\overrightarrow{i} + 2\overrightarrow{j} - \overrightarrow{k}$

15. Prove that $\nabla \times \left(\phi\vec{f}\right) = \phi\left(\nabla \times \vec{f}\right) + (\nabla\phi) \times \vec{f}$

16. Calculate Curl $\overrightarrow{f} = (x^2 + y^2 + z^2)^{-1/2}\,(yz\overrightarrow{i} + zx\overrightarrow{j} + xy\overrightarrow{k})$